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Units 3 and 4 · External exam revision

Mathematical Methods: External Game plan

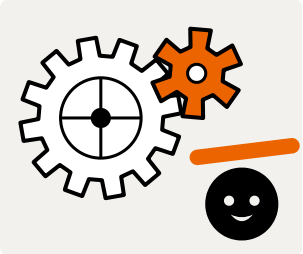
Find the right technique for every external exam question, then work it through step by step.

Inside



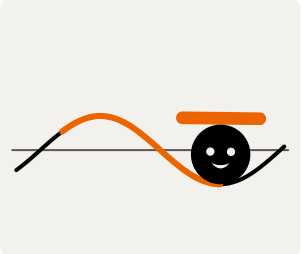
Start here

The First Question



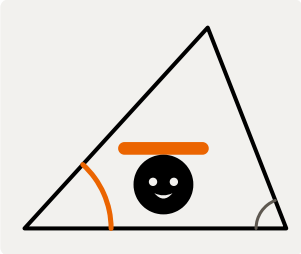
Game plan 0

The Mechanics of Methods



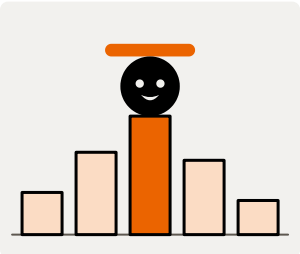
Game plan 1

Calculus



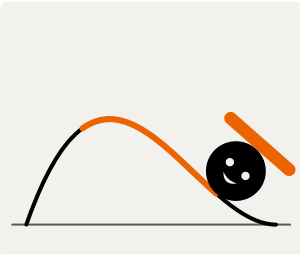
Game plan 2

Trigonometry



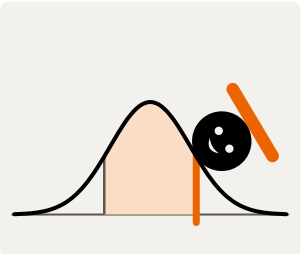
Game plan 3

Discrete random variables



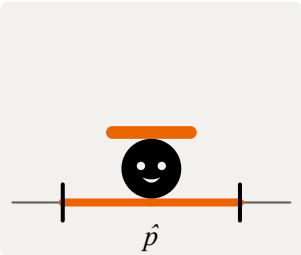
Game plan 4

Probability density functions



Game plan 5

The normal distribution



Game plan 6

Sample statistics

Name _____

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Contents

Seven game plans for the external exam. Start every question with the First Question to find the right one. Work on The Mechanics of Methods all year, because the other six assume you have it down.

? **The First Question** **3**
Ask these at the start of every question to find your game plan.

0 **The Mechanics of Methods** · Units 3 and 4 **4**
Differentiation and integration techniques, finding the equation of a graph, and the unit circle.

1 **Calculus** · Units 3 and 4 **10**
Differentiation, integration and the applied calculus techniques that build on them.

2 **Trigonometry** · Unit 4 **18**
Sine and cosine rules, area, bearings and modelling.

3 **Discrete random variables** · Unit 3 **24**
Bernoulli trials, binomial distributions and general discrete distributions.

4 **Probability density functions** · Unit 4 **30**
Continuous random variables, area, mean and variance.

5 **The normal distribution** · Unit 4 **35**
Probabilities, quantiles and z-scores.

6 **Sample statistics** · Unit 4 **39**
Sample proportions, margin of error and confidence intervals.

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Start here

The First Question

Ask yourself these every time you start an external exam question, in this order. They tell you which game plan to use. Check the statistics wording first, because some statistics questions need calculus along the way.

Step 1

Is it a statistics question?

Ask these in order and stop at the first yes.

1. Does it say "normally distributed"? → **Game plan 5**
2. Does it say "probability density function", or give a function on an interval that is zero everywhere else? → **Game plan 4**
3. Does it mention a margin of error, a confidence interval, 95% or 99%, a sample proportion or a sample size? → **Game plan 6**
4. Does it define success and failure, or say "binomial distribution"? → **Game plan 3**

Step 2 · If every answer in step 1 was no

"Am I required to differentiate or integrate in this question?"

Yes	No
Game plan 1 · Calculus	**Game plan 2 · Trigonometry**

Habit

The most common mistake I see in practice exams is reaching for the wrong technique. Answer these questions first, before you touch your calculator.

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Game plan 0

The Mechanics of Methods

The other six game plans tell you which technique a question needs. This one is the techniques themselves. Every other game plan assumes you can do these without stopping to think.

Part 1

Differentiation techniques

You need to know the standard derivatives by heart and choose the right rule for how they are put together.

Standard derivatives

$f(x)$	$f'(x)$
x^n	nx^{n-1}
e^x	e^x
$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$

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Putting them together

Chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Product rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Game plan 0.1

Choosing a rule

1 Is it a single standard function of x? Use the table and you are done.

2 Is one function inside another, like e^{3x^2} or $\sin(5x - 1)$? Use the chain rule. Differentiate the outside, keep the inside, then multiply by the derivative of the inside.

3 Are two functions of x multiplied together? Use the product rule. Write down u, v, u' and v' before you substitute.

4 Is one function of x divided by another? Use the quotient rule. The top starts with v, so the subtraction goes the right way round.

5 Inside the product or quotient rule, does u or v need the chain rule? Do that first, on the side.

6 Tidy up only as far as the question needs. Factorise if you will be setting the derivative to zero next.

> **Watch for**
>
> The two slips I see most are dropping the derivative of the inside and losing the minus sign when you differentiate $\cos x$.

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Part 2

Integration techniques

Integration is differentiation run backwards. Most of the work is getting the expression into a form you can integrate one term at a time.

Standard integrals

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

$$\int \frac{1}{x}\,dx = \ln x + c, \quad x > 0$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln(ax+b) + c$$

$$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + c$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + c$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Game plan 0.2

Integrating

1 Rewrite first. Turn roots and fractions into powers of x, expand brackets and split fractions over a single term.

2 Is there a linear expression $ax+b$ inside? Integrate as normal, then divide by a.

3 Integrate one term at a time and add $+c$ to an indefinite integral.

4 Have you been given a data point, like $f(0) = 3$? Substitute it to find c.

5 Is it a definite integral? Write the antiderivative in square brackets, then substitute the top bound and subtract the bottom one. You can leave $+c$ off.

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6 Check by differentiating your answer. You should get back what you started with.

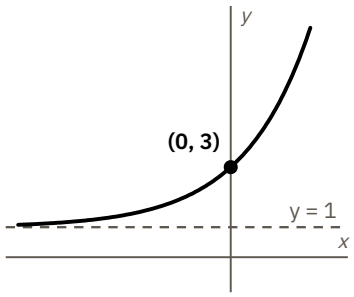
> **Watch for**
>
> With a linear expression inside, you divide by a. If you multiply by it, you have done the chain rule forwards by mistake.

Part 3

Finding the equation of a graph

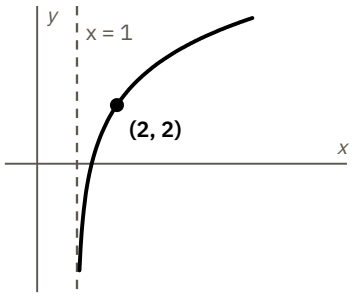
Here you are given the graph and need its equation. Every parameter in the general form shows up as a feature you can read off the graph, so the work is matching each feature to its parameter.

General form	Where to find each parameter
Exponential $y = ae^{bx} + k$	k is the horizontal asymptote. The y-intercept is $a + k$, so it gives you a. One more point gives you b.
Logarithmic $y = a\ln(x - h) + k$	h is the vertical asymptote. At $x = h + 1$ the log is zero, so $y = k$ there. One more point gives you a.
Sinusoidal $y = a\sin(b(x - h)) + k$	k is the centre line, halfway between the maximum and minimum. $\lvert a \rvert$ is half the distance from the maximum to the minimum. Read the period off the graph and use $b = \frac{2\pi}{\text{period}}$. h is where the curve crosses the centre line going up. The same goes for cosine, but h is where it reaches a maximum.



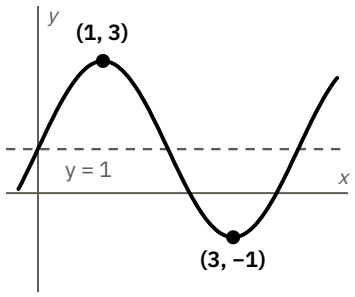
Exponential · $y = 2e^x + 1$

Asymptote $y = 1$ gives $k = 1$. The y-intercept is $a + k = 3$, so $a = 2$.



Logarithmic · $y = 2\ln(x - 1) + 2$

Asymptote $x = 1$ gives $h = 1$. At $x = h + 1 = 2$, $y = k = 2$.



Sinusoidal · $y = 2\sin\left(\frac{\pi}{2}x\right) + 1$

Centre line $y = 1$ and amplitude 2. The period is 4, so $b = \frac{2\pi}{4} = \frac{\pi}{2}$.

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Game plan 0.3

Finding the equation of a graph

1 What type of function is it? A horizontal asymptote means exponential, a vertical asymptote means log, and a repeating wave means sine or cosine. Write down the general form.

2 Read the asymptote or the centre line. That gives you k, or h for a log.

3 For a trig graph, read the maximum, the minimum and the period. Use them to find a and b.

4 Count the unknowns that are left. Substitute one point from the graph for each one, starting with the intercepts.

5 If two unknowns are left, use two points and solve the equations simultaneously. For an exponential, dividing one equation by the other often removes a.

6 Check your equation with a point you did not use, and check the direction. A negative a flips the graph.

Watch for

Read the scale on the axes before you read any values. Trig graphs are often marked in multiples of $\frac{\pi}{4}$ or in hours, and a misread period throws off b.

Part 4

The unit circle

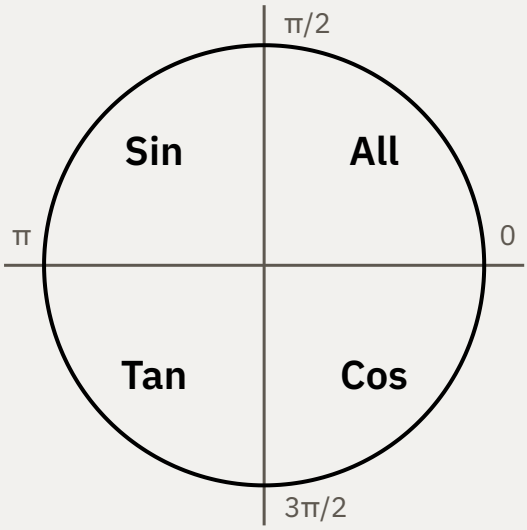
Technology-free questions expect you to know these exact values. They come up in derivatives, definite integrals and the sine and cosine rules.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1

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θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	undefined

Which ratios are positive



Going anticlockwise from the first quadrant: All Stations To Central.

Game plan 0.4

Solving a trig equation exactly

1 Rearrange so the trig ratio is on its own, for example $\sin x = -\frac{1}{2}$.

2 Ignore the sign and find the reference angle from the table. Here it is $\frac{\pi}{6}$.

3 Use the sign to pick the quadrants. Sine is negative in the third and fourth quadrants.

4 Write the angle in each quadrant: $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$ and $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$.

5 Check the domain. Add or subtract whole periods until you have every solution in it, and no more.

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Game plan 1

Calculus

Differentiation, integration and the applied calculus techniques that build on them.

Reach for it when

The question asks you to find a rate, gradient, velocity, maximum, tangent, area or total change, and step 1 of the First Question found no statistics wording. Anything that needs differentiation or integration belongs here, and it splits into two pathways.

Part 1

The two pathways

Decide first whether you are differentiating or integrating. Each pathway is a set of questions to ask yourself in order.

Game plan 1.1

Differentiation

1	Check that you need to differentiate. The question will want a rate of change.
2	What are you asked to do? Find a maximum, rate of change, velocity, gradient, or a tangent line.
3	Is your equation ready to differentiate? If it is not, find the unknown first.
4	How many times do you need to differentiate? Your answer to step 2 decides this.
5	Which differentiation rules apply here? Game plan 0.1 will help you choose.
6	What units is the question in? Give your answer in those units.

Game plan 1.2

Integration

1	Check that you need to integrate. The question will give you a rate and want a total.

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2 You are told how something changes, but you need to find how much there is.

3 What is your data point? The QCAA have to give you one so that you can find c.

4 How many times do you need to integrate? Two data points can mean you integrate twice.

5 Which integration rules apply here? Game plan 0.2 has them.

6 What units is the question in? Give your answer in those units.

Part 2

Applied calculus

These are the special cases where you use calculus to solve a problem. Each one has its own game plan.

Estimating area under curves Area under and between curves

Turning points and points of inflection Interpreting graphs with the first and second derivative

Optimisation Total change of a function

Game plan 1.3

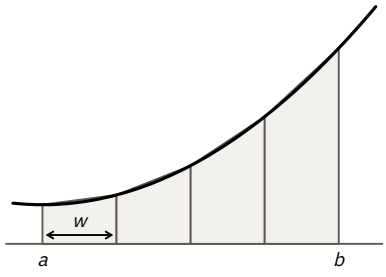
The trapezoidal rule: estimating area with strips

1 Determine the width and number of strips.

2 Determine each x_n.

3 Determine each $f(x_n)$.

4 Apply the trapezoidal rule.



Four strips of width w. The curve is concave up, so each strip pokes slightly above it, and the estimate is a little too big.

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Strip width

$$w = \frac{b-a}{n}$$

Trapezoidal rule

$$A \approx \frac{w}{2}(\text{start} + 2(\text{middle}) + \text{end})$$

Remember If your curve is concave up, the trapezoidal rule will slightly overestimate the area. If it is concave down, it will slightly underestimate it.

Game plan 1.4

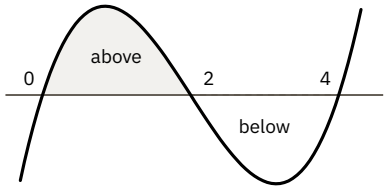
Area under a curve · Technology-free

1 Sketch the function by hand and identify the area you need.

2 Decide whether the area is above or below the x-axis. Fully above needs nothing extra. Fully below, take the absolute value of the integral. Partly both, split it and take the absolute value of the negative part.

3 Identify the bounds from the given values, or the x-intercepts.

4 Apply the Unit 3 Topic 4 integration techniques, then evaluate the definite integral.



Partly above, partly below. Split the integral at $x = 2$ and take the absolute value of the part below.

Game plan 1.5

Area under a curve · Technology-active

1 Sketch the function on your graphics calculator.

2 Deal with any area below the axis the same way as Technology-free, using the absolute value.

3 Find the bounds from the given values, or use the roots function for the x-intercepts.

4 Use the gsolv, fint or mixed function to determine the area.

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Tip The mixed function on your graphics calculator can also evaluate an area that is partly above and partly below the axis.

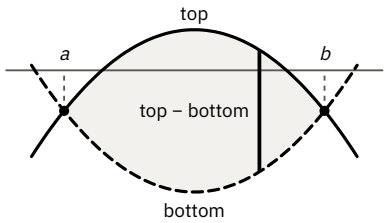
Game plan 1.6

Area between curves

1 Sketch both curves to see which is on top. Do this by hand if it is Technology-free, or on your calculator if it is Technology-active.

2 Find the points of intersection. Set the functions equal if it is Technology-free, or use the intersection tool if it is Technology-active. These become your bounds.

3 Integrate top minus bottom across those bounds.



The intersections give the bounds. Top minus bottom still works when the region dips below the x-axis.

Area between two curves

$$A = \int_a^b (f(x) - g(x))\,dx$$

Watch for

In complex unfamiliar questions you may need to define one or both functions first. The QCAA also like to give you the area and ask you to find an unknown.

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Part 3

Analysing a function

Algebraic analysis of functions uses the first and second derivatives to find and describe the key features of a graph.

Game plan 1.7

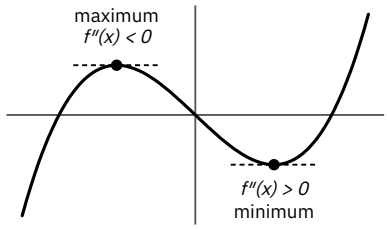
Turning point and its nature

1 Find the first derivative.

2 Set $f'(x) = 0$ to get the x-coordinate(s). Substitute back into $f(x)$ for the y-coordinate.

3 Find the second derivative $f''(x)$.

4 Substitute your x into $f''(x)$. If $f''(x) > 0$ it is concave up, so it is a minimum. If $f''(x) < 0$ it is concave down, so it is a maximum.



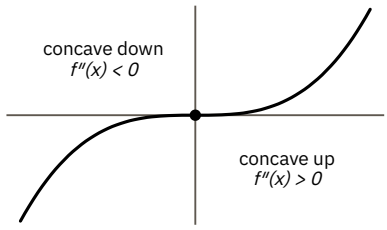
Both points have $f'(x) = 0$. The sign of $f''(x)$ tells them apart.

Game plan 1.8

Point of inflection

1 Set $f''(x) = 0$ to find the x-coordinate. Substitute back for y if you need it.

2 Use a table to check that $f''(x)$ changes sign. If it does not, it is not a point of inflection.



$f''(x)$ changes sign at the dot, so the concavity changes. That is what makes it a point of inflection.

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Interpreting graphs with the first and second derivative

	$= 0$	> 0	< 0
$f(x)$	x-intercept	Above the x-axis	Below the x-axis
$f'(x)$	Stationary point	f increasing	f decreasing
$f''(x)$	Potential point of inflection	f concave up ∪	f concave down ∩

Game plan 1.9

Reading a graph, step by step

1. **Start.** What are you given, and what do you need to find?
2. **Mark the key x-values:** intercepts, turning points and inflections.
3. **Translate.** Apply the table above to each of those x-values.
4. **Join the dots** using the signs: rising or falling, above or below, concave up or down.
5. **Check** that the sign actually changed. A zero with no sign change is not a turning point or a point of inflection.

Game plan 1.10

Optimisation

1. Identify the key information in the problem.
2. Draw a picture.
3. Construct an equation that represents the situation.
4. Express your equation in terms of one variable.
5. Set the first derivative to zero to find a potential turning point.

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6 Use the second derivative to check whether it is a minimum or a maximum.

7 Answer the question in context.

Game plan 1.11

Total change of a function

1 Identify that the function you are given is a rate function.

2 Recognise that total change is found by integrating the rate function.

3 Identify the bounds. For example, water released between 3 and 5 minutes means you integrate from 3 to 5.

4 Evaluate the definite integral of the rate across those bounds.

Part 4

Common QCAA language: Calculus

The wording of a question tells you which operation and which applied technique you need. Look for these phrases.

Core calculus language

Operation	What the question says
Differentiation	Rate of change, instantaneous rate of change, "the rate at which ... is changing", how fast ... is changing. In kinematics, velocity is the first derivative and acceleration is the second. Gradient of the tangent or curve at a point. "Increasing or decreasing by [rate] per [unit]." **Your move** Look at the units in the answer. If they are a rate, like °C/hour or members/day, differentiate.
Integration	"$\int \ldots \, dx$", "Determine $\int \ldots$", evaluate the definite integral, antidifferentiate, antiderivative. Given the rate, find the quantity, for example velocity to displacement. "Initially ..." with a rate means you find c. "Express your answer in the form $\ln(a)$" often signals a log antiderivative. **Your move** A rate goes in and a total comes out, so this is an integration question.

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Applied calculus language

Applied topic	What the question says
Estimating area under curves	"Estimate the area ... using n sums", "approximate area by the shaded rectangles", trapezoidal rule, using strips, approximate value of the definite integral. **Your move** Rectangles, strips or a table of values next to "estimate" or "approximate" tell you to add up shapes instead of integrating.
Area between and below curves	"Area between $y = \ldots$ and the x-axis from $x = a$ to $x = b$", "total enclosed area", "region bounded by ...", "state an integral expression representing the area". **Your move** When the region is enclosed, bounded or between curves and nothing says estimate, integrate for the exact area.
Turning points and inflection	"Determine the stationary or turning point", "the nature of the stationary point", "point of inflection", "show ... has a point of inflection at $x = \ldots$", the second derivative. **Your move** A named point is the target. Solve $f' = 0$ for a turning point. For an inflection, solve $f'' = 0$ and check the sign changes.
Interpreting with the first and second derivative	Increasing or decreasing (the sign of f'), concave up or down (the sign of f''), "increasing most rapidly" (the maximum of f'), "identify the graph of the second derivative". **Your move** Here you describe where the graph is increasing, decreasing or concave over an interval. One point on its own will not answer it.
Optimisation	"Determine the maximum or minimum [length, area, time]", shortest, longest, greatest, least, minimise or maximise the total, "the value of x that minimises ...". **Your move** Look for the words largest, smallest, maximum or minimum in a real context. Write the function, solve for where its derivative is 0, then check it is the one you want.
Total change of a function	"Total change in the [quantity]", distance travelled, change in displacement from $t = a$ to $t = b$, the total number or volume over the interval. **Your move** A rate is given and you need the total over time. Integrate the rate between the two times, using $\int v\,dt$ for displacement and $\int \lvert v \rvert\,dt$ for distance.

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Game plan 2

Trigonometry

Non-right-angled triangles, area, bearings and periodic modelling.

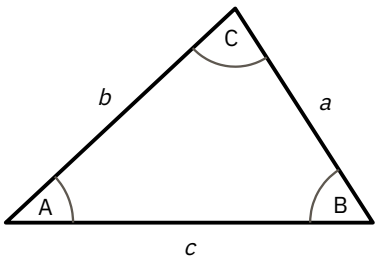
Reach for it when

The First Question found no calculus and no statistics. You will be finding sides, angles, area or bearings, or working with a periodic model. Before anything else, check your calculator is in the correct mode: degrees for triangles and bearings, radians for a periodic model.

Part 1

Choosing the rule

Look for an angle–side pair first, which is an angle and the side opposite it. If you have one, you use the sine rule. If you do not, you use the cosine rule.



Each side takes the letter of the angle opposite it. Every rule on this page uses these labels.

Game plan 2.1

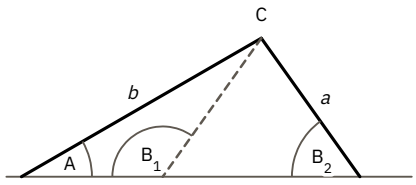
Sine rule

1. Identify the angle–side pair, the extra side and the unknown angle. Substitute them into the formula.
2. Rearrange to get $\sin\theta$ on its own.
3. Look at your value of $\sin\theta$. If it is less than 1, there may be an ambiguous case.
4. Solve for θ to find the unknown angle.
5. If it is ambiguous, subtract θ from 180° for the second case. Check that the angles work in both triangles.

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The sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Side a can land in two places, so there are two triangles. The angles at B_1 and B_2 add to 180°, which is why step 5 subtracts from 180°.

Game plan 2.2

Cosine rule

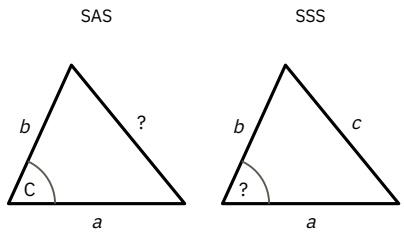
1. Identify the given sides and angles.
2. Confirm that an angle–side pair does not exist.
3. Apply the cosine rule to find the required side or angle.

The cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Rearranged for an angle (SSS)

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



With SAS you find the third side. With SSS you find an angle, using the rearranged form.

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Part 2

Area, bearings and complex problems

Each of these builds on the sine and cosine rules from Part 1.

Game plan 2.3

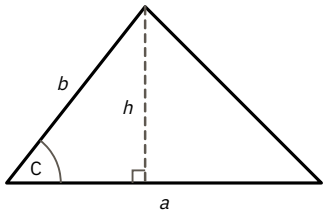
Area of a triangle

1 Find an angle between two known sides. If there is not one, find unknowns until you have one.

2 Apply the area formula.

Area from two sides and the included angle

$$A = \frac{1}{2}ab\sin C$$



The height is $h = b\sin C$, so the area is $\frac{1}{2} \times a \times b\sin C$.

Game plan 2.4

Bearings

1 Draw a diagram if none is given. Mark a north line (a vertical arrow) at each relevant point.

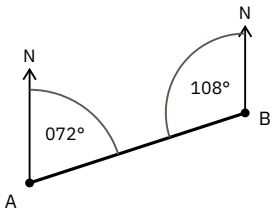
2 Convert each bearing into an angle measured clockwise from the north line at its point.

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3 Use parallel line facts to fill in the angles inside the triangle. Co-interior angles add to 180° and alternate angles are equal.

4 Once you have an angle–side pair, two sides and the included angle, or all three sides, apply the sine or cosine rule as normal.

5 Give a bearing as three figures measured clockwise from north, for example 072°T. Use the back bearing ($\pm 180^\circ$) to see it from the other point.



The north lines are parallel, so the two marked angles are co-interior and add to 180°. The bearing of A from B is the back bearing, $072^\circ + 180^\circ = 252^\circ$T.

Game plan 2.5

Complex modelling

In a complex unfamiliar question, the triangle is usually missing the piece you need, or the trigonometry is only the first half of the question.

1 Draw and fully label a diagram. If none is given, the diagram can earn a mark of its own. Let a variable stand for the unknown, such as h, x or θ.

2 Find the triangle that holds the unknown. In 3D, each side may lie on a different face, so use Pythagoras on each face to write the sides in terms of your variable.

3 Look for what you were not told. Parallel lines give alternate angles, and similar triangles give a ratio of sides.

4 Apply the sine rule, cosine rule or area formula, even with the variable still in it. With three sides and no angle, use the cosine rule backwards to find the angle first.

5 Let the last instruction finish it. You might solve for the variable, differentiate to optimise, or integrate for an area.

6 Re-read the question and answer exactly what it asks. A justified decision needs your value compared with the requirement, with units.

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Part 3

Common QCAA language: Trigonometry

The wording tells you the context and the rule you need. Look for these phrases.

Note Trigonometry is different to the other topics. The language tells you the context, but I want you to look just as closely at what you are given in the triangle.

Question type	What the question says and gives you
Sine rule	"The length of side …", "the size of angle …", "how far", "the distance from …", often after a bearing or elevation set-up. You are given an angle and the side opposite it, plus one more piece. **Your move** Know an angle and the side across from it? Reach for the sine rule.
Cosine rule	"The length of the side …" (given the two sides either side of an angle), "the size of angle …" (given all three sides). You do not have an angle with its opposite side. You have SAS (find the third side) or SSS (find an angle). **Your move** Two sides with the angle between them, or all three sides, point you to the cosine rule.
Area of a triangle	"The area of the triangle / paddock / region / plot / block". You need two sides and the angle between them for $\frac{1}{2}ab\sin C$. Find the angle first if it is missing. **Your move** Area of a triangle with two sides and the angle between them? Use $\frac{1}{2}ab\sin C$.
Bearings	"Bearing of B from A", "052°T" or "true bearing", "N40°E", "due north / south", "the direction", and also angle of elevation or depression. **Your move** Compass directions or the word bearing mean a bearings question. Draw a north line at each point, use the sine or cosine rule, and write the bearing with three figures.
Complex modelling	"Make a justified decision", "does it meet the requirement", "minimise" or "maximise" with a triangle, "determine the mass" or "the cost", a 3D object such as a container or tower, parallel walls or fences, or no diagram given. **Your move** A variable in the triangle, or a final step that asks you to decide, optimise or integrate, makes this complex modelling. Do the trigonometry first, then read the last instruction to see which topic finishes the job.

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Question type	What the question says and gives you
Periodic modelling	"Modelled by $R(t) = a\sin(b(t-h)) + k$" (or cos), "amplitude", "period", "hours after midnight", tides, temperature, height, Ferris wheel. **Your move** Something real that repeats, like tides or temperatures, needs a periodic model. Write the sine or cosine model first. The next instruction (solve, differentiate or integrate) tells you what comes after.

To find the equation of a periodic graph, use Game plan 0.3.

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Game plan 3

Discrete random variables

Bernoulli trials, binomial distributions and general discrete distributions.

Reach for it when

The First Question found statistics, and the question defines success and failure, says "binomial distribution", or gives more than two outcomes in a data table. A single trial is Bernoulli. Repeated trials are binomial.

You have a Bernoulli trial when success is clearly defined and the probability of each outcome is fixed. If the trial is repeated, you are in the binomial game plan.

Part 1

Bernoulli and binomial

Both have exactly two outcomes on each trial, success and failure. A Bernoulli variable is one trial. A binomial variable counts the successes over n trials.

Game plan 3.1

Bernoulli distributions

1 Identify the probability of success, p, and of failure, $1-p$.

2 Apply the mean, variance or standard deviation formula as asked.

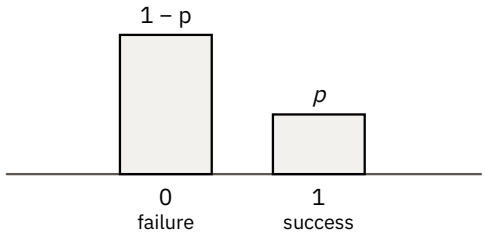
Mean, variance and standard deviation

Mean	Variance	Standard deviation
$E(X)=\mu=p$	$\text{Var}(X)=p(1-p)$	$\text{SD}(X)=\sqrt{\text{Var}(X)}$

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The Bernoulli distribution

x	0	1
$P(X = x)$	$1 - p$	p



With $p = 0.3$, the bar at 1 has height p and the bar at 0 has height $1 - p$. The two bars add to 1.

Game plan 3.2

Binomial distributions

1 Identify that the problem is binomial.

2 Identify the number of trials n, the required successes r, the probability of success p and of failure $1 - p$.

3 For the mean, variance or standard deviation, apply the formulas below.

4 For a probability, first define it carefully from the language cues.

- more than $r \rightarrow P(X > r)$
- at least or no less than $r \rightarrow P(X \geq r)$
- less than $r \rightarrow P(X < r)$
- at most or no more than $r \rightarrow P(X \leq r)$
- exactly $r \rightarrow P(X = r)$

5 Technology-free: apply the formula. It is often quicker to find the probability of the event not happening and subtract it from 1.

6 Technology-active: use BPD for a $P(X = r)$ situation and BCD for anything else.

7 If you are given the probability, work backwards to find the unknown using the $P(X = r)$ formula.

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Mean and variance

Mean

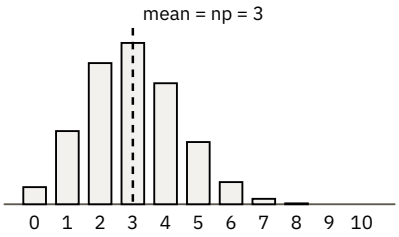
$$E(X) = np$$

Variance

$$\text{Var}(X) = np(1-p)$$

Exact probability (Technology-free)

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$$



$X \sim \text{Bin}(10,\ 0.3)$. The bars are tallest around the mean, $np = 3$.

Binomial coefficients

Definition

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Useful identities

$$\binom{n}{0} = \binom{n}{n} = 1$$

$$\binom{n}{1} = \binom{n}{n-1} = n$$

$$\binom{n}{2} = \frac{n(n-1)}{2}$$

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Quantity words → probability

Phrase in the question	Probability required
more than	$P(X > r)$
at least · no less than	$P(X \geq r)$
less than · fewer than	$P(X < r)$
at most · no more than	$P(X \leq r)$
exactly	$P(X = r)$

Part 2

General discrete random variables

If there are more than two outcomes in a table or data display, you have a general discrete random variable.

Game plan 3.3

General discrete random variables

1 If there are more than two outcomes in a table or data display, you have a discrete random variable.

2 To show a table is a valid distribution, check both: the probabilities add to 1, and each probability is between 0 and 1.

3 For an unknown value in a table, use the fact that the probabilities must add to 1.

4 For the expected value, multiply each value of X by its probability and add.

5 For the variance, apply the formula below. Find $E(X^2)$ by squaring your values of X, multiplying each by its probability and adding.

6 For the standard deviation, take the square root of the variance.

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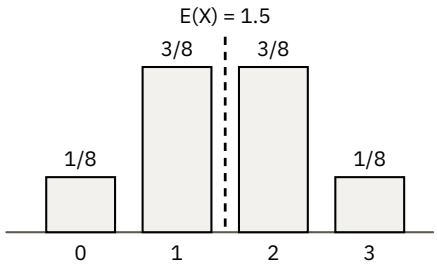
Variance of a discrete random variable

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

Example · A valid distribution

x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

The probabilities add to 1 and each one lies in $[0, 1]$, so this is a valid distribution.



The distribution is symmetric, so $E(X) = 1.5$ sits halfway between the two tallest bars.

Part 3

Common QCAA language: Discrete random variables

The wording tells you which distribution you have and what to find. Look for these phrases.

Bernoulli and binomial cues

Question type	What the question says
Identify a Bernoulli trial	"Bernoulli experiment / trial / distribution", a single action with two outcomes (success / failure, hit / miss, four / not a four). **Your move** One trial with two outcomes, and the word "Bernoulli". All you need is p and $p(1-p)$.
Justify a binomial model	"State the conditions that make this suitable for a binomial random variable", "which context is (not) suitable for a binomial model". **Your move** List all four conditions: a fixed n, independent trials, two outcomes, and the same p each time.
Mean, variance and SD	"The mean / expected number", "the variance", "the standard deviation" of the distribution. **Your move** For the mean, variance or SD of a binomial, the formulas are np and $np(1-p)$.

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Question type	What the question says
Binomial probability	"The probability that ... [exactly / at least / at most / no more than / fewer than / more than / between] are successful". **Your move** Change words like "at least" or "fewer than" into $\geq$ or $<$ first. Then use the formula or your calculator.
Find a missing n or p	A probability is given and "determine n" or "determine p". **Your move** The probability is given but n or p is unknown. Set up the binomial formula and solve for the missing value.

General discrete random variable cues

Question type	What the question says
Properties of a distribution	"Show that this is a valid probability distribution", "determine the value of k so the table represents a probability distribution". **Your move** Check two things: every probability is between 0 and 1, and together they add to 1.
Construct a distribution	"Construct / complete the probability distribution table for X", "use the spinner / dice / histogram to determine the distribution". **Your move** Find every probability from the information, then confirm the total is 1.
Mean, variance and SD	"Determine the expected value $E(X)$", "the variance $\text{Var}(X)$", "the standard deviation", applied as "the expected winnings / number / cost". **Your move** Use $\sum x\,p(x)$ and $\sum x^2 p(x)$, then $\text{Var}(X) = E(X^2) - (E(X))^2$.
Modelling with DRVs	"The expected profit / winnings", "is the game fair", "in the long run ...", a decision between options. **Your move** Calculate the expected value first. The question then wants you to use it in context.

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Game plan 4

Probability density functions

Area, probability, mean and variance for continuous random variables.

Reach for it when

The First Question found statistics, and the question uses the phrase "probability density function", or you recognise its general form. This is a function defined over an interval and zero everywhere else.

Part 1

The game plan, in order

Work through these four questions in order. The first two come before anything else, because an unknown constant has to be found before you can use the PDF.

General form of a PDF

$$p(x) = \begin{cases} f(x), & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

Game plan 4.1

PDFs, in order

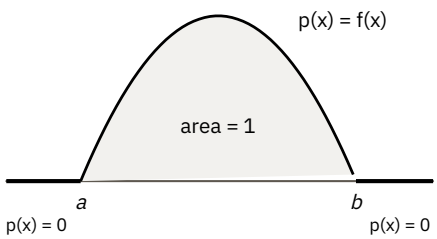
1 Check that it is a PDF question, from the phrase or the general form above.

2 Are there unknown values in the PDF? Every PDF has an area of 1 between its bounds. Integrate over that interval and set it equal to 1 to find them.

3 Are you asked for a probability between two values? Integrate, using those values as bounds.

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4 Are you asked for the expected value, variance or standard deviation? Use the formulas in Game plan 4.4.



Between a and b the PDF is $f(x)$, and the total area under it is 1. Everywhere else it is zero.

Part 2

The three calculations

Each one is a definite integral of $p(x)$, or of $p(x)$ multiplied by something.

Game plan 4.2

Finding an unknown constant

1 Every PDF has an area of 1 between its bounds.

2 Integrate over that interval and set it equal to 1.

3 Solve for the unknown.

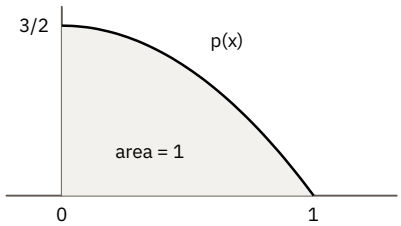
Worked example

$p(x) = c(1 - x^2)$ for $0 \leq x \leq 1$

$$\int_0^1 c(1 - x^2)\,dx = 1$$

$$c\left[x - \tfrac{x^3}{3}\right]_0^1 = \tfrac{2c}{3} = 1$$

$$c = \tfrac{3}{2}$$



With $c = \frac{3}{2}$ the curve starts at height $\frac{3}{2}$, and the shaded area is exactly 1.

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Game plan 4.3

Probability between values

1 Use the two values as the bounds.

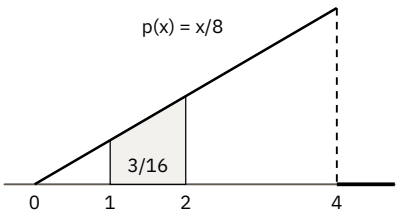
2 Integrate $p(x)$ between them. A probability is the shaded area under $p(x)$ between the bounds.

Worked example

$p(x) = \frac{1}{8}x$ for $0 \leq x \leq 4$

$$P(1 \leq X \leq 2) = \int_1^2 \frac{1}{8}x\,dx$$

$$= \left[\frac{x^2}{16}\right]_1^2 = \frac{4}{16} - \frac{1}{16} = \frac{3}{16}$$



$P(1 \leq X \leq 2)$ is the shaded area between the two bounds.

Game plan 4.4

Mean, variance and standard deviation

1 Find the expected value, $E(X) = \int x\,p(x)\,dx$, over the bounds of the PDF.

2 For the variance, find $E(X^2) = \int x^2 p(x)\,dx$, then use $\text{Var}(X) = E(X^2) - (E(X))^2$.

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3 Are you asked for the standard deviation?
Take the square root of the variance.

Formulas

Expected value

$$E(X) = \int_{-\infty}^{\infty} x\,p(x)\,dx$$

Variance

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

Worked example

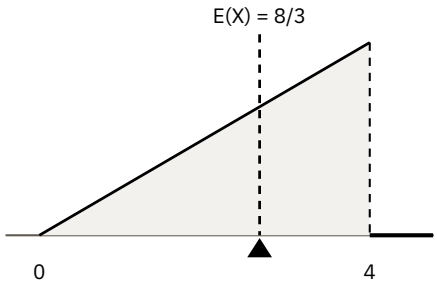
$p(x) = \frac{1}{8}x$ for $0 \leq x \leq 4$

$$E(X) = \int_0^4 x \cdot \tfrac{1}{8}x\,dx = \left[\tfrac{x^3}{24}\right]_0^4 = \tfrac{8}{3}$$

$$E(X^2) = \int_0^4 x^2 \cdot \tfrac{1}{8}x\,dx = \left[\tfrac{x^4}{32}\right]_0^4 = 8$$

$$\text{Var}(X) = 8 - \left(\tfrac{8}{3}\right)^2 = \tfrac{8}{9}$$

$$\text{SD}(X) = \sqrt{\tfrac{8}{9}} = \tfrac{2\sqrt{2}}{3} \approx 0.943$$



$E(X) = \frac{8}{3}$ is the balance point of the shaded region. It sits right of the middle because more area is on the right.

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Part 3

Common QCAA language: Probability density functions

The QCAA will usually tell you it is a PDF with the phrase "probability density function". If they do not, look for the general form: $p(x) = f(x)$ on $a \leq x \leq b$, and zero otherwise.

Question type	What the question says
Parameter recovery	"Such that $p(x)$ is a valid probability density function", "determine the value of k / a", "find the constant". **Your move** An unknown constant comes first. Integrate the whole PDF over its interval, make it equal 1, and solve.
Probability	"The probability that X is between a and b / less than / more than / at least", "$P(a < X < b)$". **Your move** A probability between two values is a definite integral of $p(x)$. With a CDF, it is $P(a < X < b) = F(b) - F(a)$.
Mean, variance and SD	"The mean / expected value / $E(X)$", "the variance / $\text{Var}(X)$", "the standard deviation". **Your move** Put an extra x inside the integral to get the mean. Get the variance from $\int (x-\mu)^2 p(x)\,dx$ or $\int x^2 p(x)\,dx - \mu^2$, then square root it for the SD. $E(X)$ comes up most often.
Percentile, median, quartiles	"The median", "the nth percentile", "the lower / upper quartile", and "the mode / most likely value". **Your move** For a median, quartile or percentile, integrate up to the unknown value, make it equal the proportion, and solve. For the mode, find where $p(x)$ has its maximum.

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Game plan 5

The normal distribution

Probabilities, quantiles and recovering unknown parameters with z-scores.

Reach for it when

The First Question found statistics, and the question uses the phrase "normally distributed". It will not ask you to differentiate or integrate.

Part 1

Question types

A probability is an area under the curve. A quantile works backwards from a given area to an x value, x_0. If you are not given the mean or standard deviation, use a z-score to find it first.

Model

$$X \sim N(\mu, \sigma^2)$$

Standardising

$$z = \frac{x - \mu}{\sigma}$$

Given μ and σ, find a probability	Game plan 5.1
Given μ and σ, find a quantile	Game plan 5.2
μ or σ unknown, recover it with a z-score	Game plan 5.3

Complex unfamiliar questions often combine a normal distribution with a binomial. Use the normal distribution to find p, then carry it into Game plan 3.2.

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Part 2

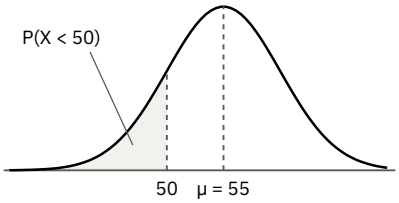
The three game plans

Each one starts the same way: identify the mean and standard deviation, then draw a picture. The picture is what stops you choosing the wrong tail.

Game plan 5.1

Finding a probability

1. Identify the mean and standard deviation.
2. Identify the given value and draw a picture.
3. Use the picture to identify the upper and lower bounds.
4. Solve. Use the empirical rule if Technology-free, or the NCD function if Technology-active.

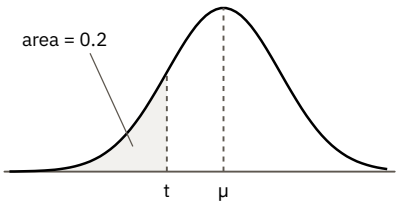


The picture gives the bounds. Here the upper bound is 50 and there is no lower bound, so use a very large negative number.

Game plan 5.2

Finding a quantile

1. Identify the mean and standard deviation.
2. Identify the given probability and draw a picture.
3. Use the picture to decide whether the tail is left or right.
4. Solve. Use the empirical rule if Technology-free, or the invNorm function if Technology-active.



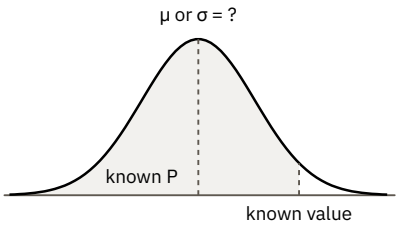
The area is a left tail, so t sits below the mean. For a right tail of 0.2, the area to the left is 0.8.

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Game plan 5.3

μ or σ unknown

1. Identify the known values. The extra piece of information gives you an x value and its probability.
2. Calculate the z-score from that extra information.
3. Substitute into the z-score formula to find the unknown mean or standard deviation.
4. Answer the question in context.



The value and its probability give you z. Then the z-score formula has only one unknown.

Part 3

Common QCAA language: Normal distributions

Look for "normally distributed", "approximately normally distributed", "modelled by a normal distribution", $X \sim N(\mu, \sigma^2)$ or "standard normal". You will usually see a real-world measurement, such as height, mass, time, volume, IQ or load.

Question type	What the question says
Finding a probability	"Determine the probability that ... is between / more than / less than / at least", "what proportion / percentage are above / below / between", "the expected number that exceed ...". The key words are between, more than, less than, within and exceeds. **Your move** Mean, SD and a value are given, and you need a probability. That is normal CD: NCD(lower, upper, μ, σ).
Finding a quantile	"The top 15% / bottom 10%", "taller than exactly 90% of ...", "the smallest / largest value accepted", "the cut-off / threshold / mark needed", "the n th percentile". **Your move** Mean, SD and a probability are given, and you need the value. That is invNorm(area, μ, σ). Sketch the curve so you shade the right tail.

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Question type	What the question says
μ or σ unknown	"Given [value] corresponds to [probability] … determine the mean / standard deviation", "95% of the area lies between [two values] … determine σ", a z value given to you. **Your move** One of μ or σ is unknown, but you know a value and its probability. Turn the probability into z, put it in $z = \frac{x-\mu}{\sigma}$ and solve.

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Game plan 6

Sample statistics

Sample proportions, margin of error and confidence intervals.

Reach for it when

The First Question found statistics, and the question mentions a confidence interval, sample proportion, margin of error or sample size.

Part 1

Question types

There are two families of question. Sample proportion questions are about how $\hat{p}$ changes from sample to sample. Confidence interval questions use one sample to estimate p.

Sample proportions

Analyse bias in sampling.	Bias rules
Determine the mean and standard deviation of $\hat{p}$.	Game plan 6.1
Determine the sample size from the mean and standard deviation.	Game plan 6.1

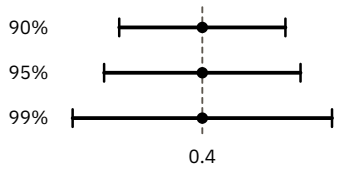
Confidence intervals

Interpret a confidence interval.	QCAA language
Calculate one from $\hat{p}$, n and a z-score.	Game plan 6.1
Solve for unknowns using the margin of error formula.	Game plan 6.1
Model with confidence intervals.	Game plan 6.1

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Interpreting or evaluating bias

- Random sampling is best, then systematic, then self-selected.
- Increasing the sample size makes the margin of error smaller.
- Increasing the level of confidence makes the margin of error larger, and lowering it makes the margin of error smaller.



The same sample at 90%, 95% and 99% confidence. More confidence means a bigger z, so a wider interval.

Part 2

Calculating a value

Every calculation uses one of three formulas. Most of the work is deciding which one the question wants.

Game plan 6.1

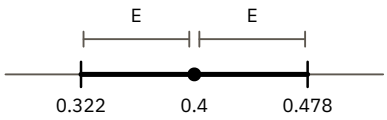
Calculating a value

1 Identify what you are given: the sample proportion $\hat{p}$, the sample size n, and the confidence level with its z-score.

2 Decide what you are asked for, then use the matching formula.

3 You can also find a confidence interval with technology: MENU → STAT → INTR → Z → 1-PROP.

4 You may need to work backwards to find an unknown, often n, from the margin of error or confidence interval formula.

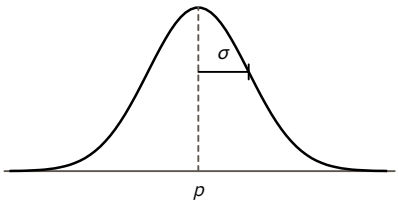


The interval is $\hat{p} \pm E$. Here $\hat{p} = 0.4$ and $n = 150$ at 95% confidence, so $E \approx 0.078$.

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Confidence level → z

90%	$z = 1.65$
95%	$z = 1.96$
99%	$z = 2.58$



Sample proportions centre on p. A bigger n makes $\sigma_{\hat{p}}$ smaller, so the curve is narrower.

Mean and SD of $\hat{p}$

$$\mu_{\hat{p}} = p \qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Margin of error

$$E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Confidence interval for a proportion

$$(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}})$$

Part 3

Common QCAA language: Sample statistics

Question type	What the question says
Analysing bias in samples	"Which sampling method would best minimise bias", "most appropriate / fairest sampling method". **Your move** The fairest method gives every member the same chance of being picked. That is usually the random number generator.
Mean and SD of $\hat{p}$	"The distribution of the sample proportion has a mean of ... and a standard deviation of ...", "determine the mean and standard deviation of $\hat{p}$". **Your move** This is about how $\hat{p}$ behaves across samples. Its mean is p and its SD is $\sqrt{p(1-p)/n}$.

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Question type	What the question says
Sample size from mean and SD	The mean and SD of $\hat{p}$ are given, with "determine the sample size" or "the value of n". **Your move** Both the mean and SD of $\hat{p}$ are given, and you need n. Solve the SD formula for n.
Interpreting a confidence interval	"Which statement correctly interprets this", "can be X% confident that the proportion …", "what does 95% confidence mean". **Your move** The right statement talks about taking many samples. Cross out any that say there is a probability that p is in this one interval.
Calculating a confidence interval	"Determine the approximate confidence interval for the proportion of …", with n, $\hat{p}$ and a confidence level (or z) supplied. **Your move** Everything is given: n, $\hat{p}$ and z. Put them straight into the formula.
Solving for an unknown	"Determine the margin of error", "the lower / upper end of the interval is …", "a margin of error of 3% … determine n", "if the confidence level increases from 95% to 99% …". **Your move** One value is missing. Rearrange the formula to get it.
Modelling with confidence intervals	An applied context with "evaluate the reasonableness of the claim", "is the claim justified", "how large a sample is required to …" or "compare the intervals". **Your move** Work out the interval first. Then see whether the claim fits inside it, or solve for the sample size you need.