LUI | | | | | | | | | | |

School code | | | | |

School name

Given name/s

Family name

Attach your barcode ID label here

Book | | of | | books used

External assessment 2026

Question and response book

Specialist Mathematics

Paper 1 — Technology-free

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are **not** permitted.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions
- Predicted paper written by Tangent Tuition. Not QCAA material.

Clear zone — margin trimmed off after completion of assessment

THIS PAGE WILL NOT BE MARKED

Section 1

Instructions

- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- Choose the best answer for Questions 1–10.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Ensure you have filled an answer bubble for each question.

Do not write outside this box.

Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 50 marks.

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QUESTION 11 (4 marks)

Consider the polynomial $P(z) = z^3 - 3z^2 + 7z - 5$, where $z \in C$.

a) Use the factor theorem to show that $(z-1)$ is a factor of $P(z)$. *[1 mark]*

b) Determine the quadratic factor $Q(z)$ such that $P(z) = (z-1)\,Q(z)$. *[1 mark]*

c) Use your result from Question 11b) to determine the solutions of $P(z) = 0$. *[2 marks]*

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QUESTION 12 (5 marks)

Consider the proposition

$$\sum_{r=1}^{n} r2^r = (n-1)2^{n+1} + 2$$

Prove that this proposition is true $\forall n \in Z^+$ using mathematical induction by completing the steps of the proof as indicated.

a) Initial statement: *[1 mark]*

Assuming the proposition is true for $n = k$,

$$\sum_{r=1}^{k} r2^r = (k-1)2^{k+1} + 2.$$

b) Inductive step: *[3 marks]*

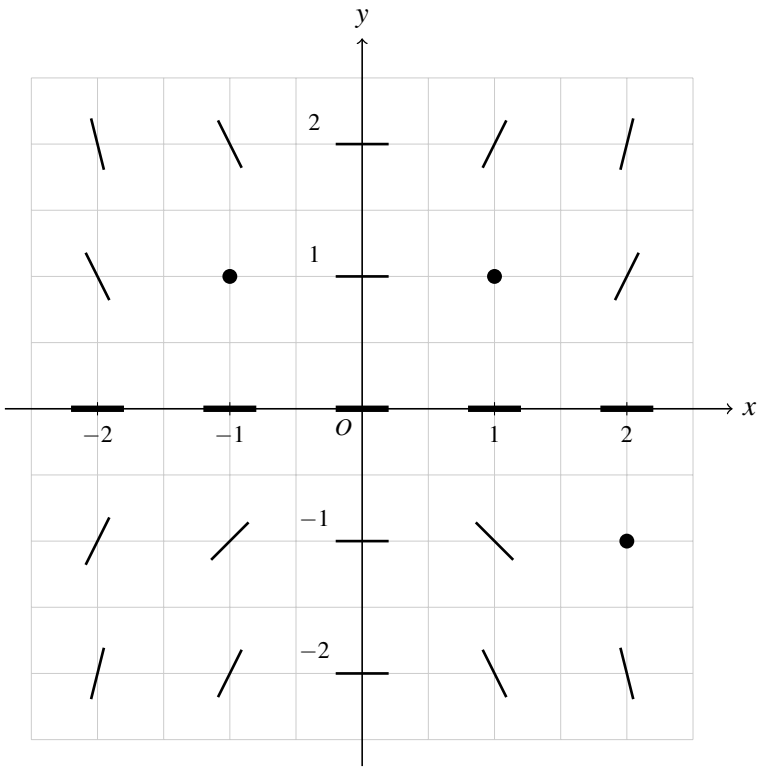
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c) Conclusion: *[1 mark]*

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QUESTION 13 (6 marks)

A slope field for the differential equation $\frac{dy}{dx} = xy$ is shown. Line segments have not been drawn at the three points marked with a dot.



a) Complete the slope field by sketching the slopes at the three points indicated. *[1 mark]*

b) Use the completed slope field to sketch the solution curve for $\frac{dy}{dx} = xy$, given the point $(0, 1)$ lies on this curve. *[1 mark]*

Note: If you make a mistake in the slope field, cancel it by ruling a single diagonal line through your work and use the additional response space on page 22 of this question and response book.

Do not write outside this box.

c) Use separation of variables to show that the general solution of the differential equation is $y = Ae^{\frac{x^2}{2}}$, where A is a constant. *[3 marks]*

d) Determine the equation of the solution curve sketched in Question 13b). *[1 mark]*

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QUESTION 14 (4 marks)

In this question, $x > 0$.

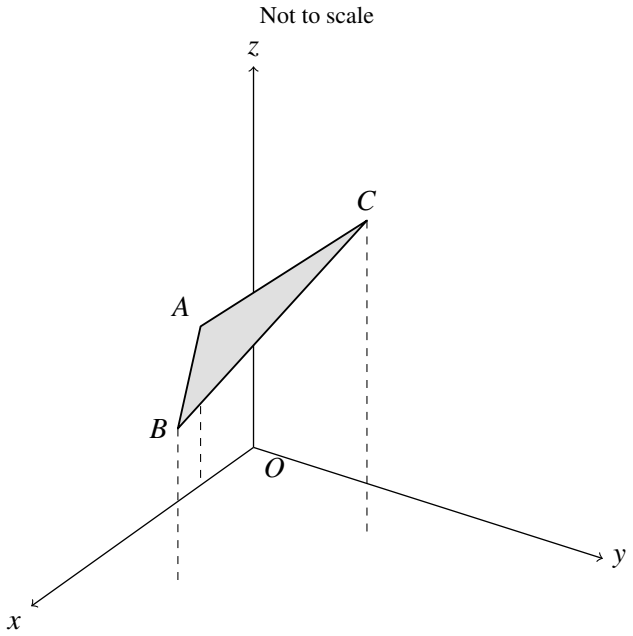
a) Express $\dfrac{1}{x(x+1)}$ in partial fractions. *[1 mark]*

b) Use integration by parts and your result from Question 14a) to determine $\displaystyle\int \frac{\ln(x)}{(x+1)^2}\,dx$. *[3 marks]*

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QUESTION 15 (7 marks)

A flat triangular shade sail is fixed at three anchor points $A(1, 0, 2)$, $B(3, 1, 2)$ and $C(1, 2, 4)$. Coordinates are measured in metres from an origin O at ground level, and the z-axis points vertically upwards.



a) Determine $\overrightarrow{AB}$ and $\overrightarrow{AC}$. *[1 mark]*

b) Determine $\overrightarrow{AB} \times \overrightarrow{AC}$. *[1 mark]*

c) Use your result from Question 15b) to determine the Cartesian equation of the plane that contains the shade sail. *[2 marks]*

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d) Use your result from Question 15b) to determine the area of the shade sail. *[1 mark]*

The top of a straight support pole touches the shade sail.

The pole lies along the line $\boldsymbol{r} = 4\hat{\boldsymbol{i}} + 3\hat{\boldsymbol{j}} + t\left(-4\hat{\boldsymbol{i}} - 4\hat{\boldsymbol{j}} + 5\hat{\boldsymbol{k}}\right)$, where t is a scalar.

e) Determine the coordinates of the point where the pole touches the shade sail. *[2 marks]*

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QUESTION 16 (6 marks)

Given $\sin(\theta) \neq 0$, use De Moivre's theorem to prove the identity

$$\frac{\sin(4\theta)}{\sin(\theta)} = 8\cos^3(\theta) - 4\cos(\theta)$$

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Do not write outside this box.

QUESTION 17 (6 marks)

A particle moves in a straight line so that its acceleration (m s^{-2}) is given by $a = x + 1$, where x is its displacement (m) from the origin O at time t seconds.

Initially, the particle is at O and has a velocity of $1\ \text{m s}^{-1}$.

Determine the time at which the particle has a speed of $5\ \text{m s}^{-1}$.

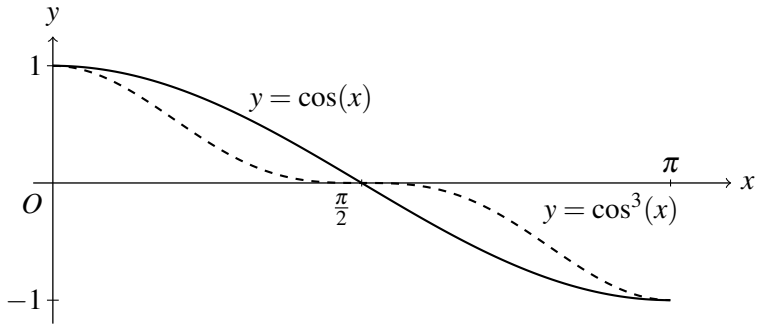
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QUESTION 18 (6 marks)

The graphs of $y = \cos(x)$ and $y = \cos^3(x)$ are shown for $0 \leq x \leq \pi$. The total area of the regions bounded by the two graphs and the line $x = k$, where $0 < k < \pi$, is $\frac{5}{8}$ units^2.

Not to scale



Determine the value of k.

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Do not write outside this box.

QUESTION 19 (6 marks)

The three cube roots of a complex number w are plotted on an Argand plane and then joined by straight lines to form a triangle with an area of $3\sqrt{3}$ units^2.

One of the cube roots of w has equal real and imaginary parts.

Determine all possible values of w. Leave your answers in Cartesian form.

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END OF PAPER

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ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Write the question number you are responding to.

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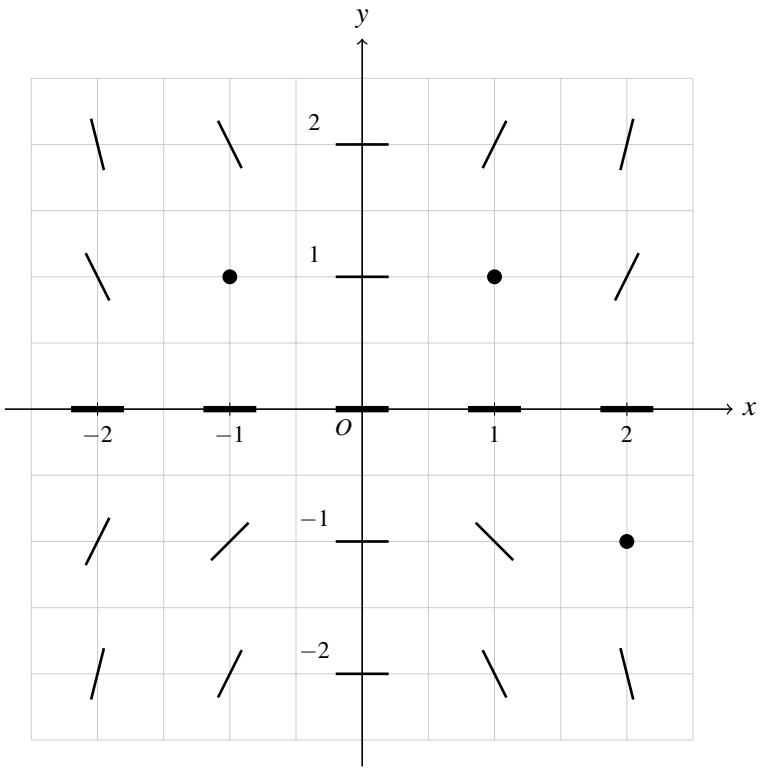
ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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ADDITIONAL RESPONSE SPACE FOR QUESTIONS 13a) AND 13b)

If you want this slope field to be marked, rule a single diagonal line through the slope field on page 6.



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